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Robust Glycemic Regulation via Takagi–Sugeno Explicit Model Predictive Control for Type 1 Diabetes Management

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Abstract

In this paper, we propose a TS-eMPC control strategy for blood glucose regulation in patients with type 1 diabetes using artificial pancreas technology. In the proposed methodology, the optimal control policy associated with MPC is computed offline using the exact fuzzy state feedback controller framework of Takagi- Sugeno form; this reduces the online complexity to O(n) without having to solve an online optimization problem. Our physiological model will be the Bergman minimal model whose only nonlinearity can be represented exactly using sector nonlinearity theory, hence obtaining a two-rule T-S system. The local gains are calculated by solving a constrained quadratic regression problem on a database generated by optimal MPC policies, with actuator saturation constraints enforced automatically. Formal closed-loop stability is demonstrated using the spectral radius of the product-norm operator on an invariant set containing initial states. Simulation results show trajectory tracking with less than 6% error compared to the optimal MPC control under meal disturbances; this corresponds to a time in range of 99% with no hypoglycemia events occurring; the performance beats all other compared MPC approaches.

I. Introduction

Diabetes mellitus type 1 is a long-term condition where the body's immune system attacks and eliminates the pancreatic beta cells responsible for producing insulin. Consequently, the condition forces an individual to regulate his or her blood sugar levels through exogenous sources of insulin [1]. Failure to control the condition leads to several health risks, including hypoglycemia and ketoacidosis, among others. The development of artificial pancreas systems seeks to automate the process through feedback control between glucose level sensors and insulin pump devices [2, 3].

Controller design for these systems is especially difficult. The glucose-insulin system exhibits nonlinearity, great variability between individuals and overtime within each individual, and constant interference through factors like food intake, exercise, hormone secretion due to stress, and exhaustion [4, 5]. Subcutaneous delivery of the insulin results in a large delay in its absorption. An efficient solution must observe strict physiological limits, namely that the amount of insulin delivered cannot be less than zero and prevent any overdosing while executing in real-time on a wearable device [6].

Model predictive control techniques have emerged as the most used technique for artificial pancreas devices due to their inherent ability to perform multi-step predictions and reference tracking [6]. Each time step involves solving an optimization problem over a fixed time horizon. For complex physiological models and extended prediction horizons, the computational load may become too high for embedded devices, leading researchers to seek approximate solutions [7, 8].

A few approaches have been considered to overcome this problem. Explicit MPC uses off-line computations of the optimal solution as a piece-wise affine function over the state space; however, the number of partitions increases exponentially with increasing model size, rendering the approach unrealistic [7]. The use of Taylor series linearization simplifies online computations but comes at the expense of stability when there is a wide range of parameter changes [7]. Event triggered MPC reduces the number of optimizations run by about half but keeps online computations [9]. Neural network approximations show high approximation quality but offer no formal stability guarantees [10, 8]; this is particularly important for safe certification of medical devices.

Takagi-Sugeno fuzzy systems provide an alternative structural approach [11]. For any smooth nonlinear system, it is possible to constructively represent it as a convex sum of local linear systems using a set of interpretable premise variables, where the convex sum coefficients are functions of the premise variables [12, 13, 14]. Applied to glycemic control, Takagi-Sugeno controllers have shown improvements over classical PID strategies [15, 16]; however, they lack a predictive horizon and show degraded performance under large meal disturbances above 60 g of carbohydrates [16, 17].

The approach proposed in this paper, referred to as TS-eMPC, combines the offline pre-computation philosophy of explicit MPC with the structural simplicity of Takagi-Sugeno representations. The optimal MPC law is approximated entirely offline by a fuzzy state-feedback law, reducing online execution to linear complexity in the state dimension while preserving predictive performance. Crucially, unlike neural surrogates, this framework admits formal stability analysis through non-quadratic Lyapunov functions and relaxed conditions [13, 18], which is essential for a safety-critical application. The Bergman minimal model serves as the physiological plant [19], and the performance of the proposed controller is evaluated through simulation and benchmarked against state-of-the-art MPC-based strategies, as summarized in Table 2.

II. Methods

a. Bergman Minimal Model

In the design of the controller, a reduced model of the physiological system for type 1 diabetes is used. The Bergman minimal model captures the core glucose-insulin dynamics through three interacting compartments, described by the following equations:

$$\dot{G}(t) = -P_G \cdot G(t) - Q(t) \cdot G(t) + P\_G \cdot G_0 + U_g(t) \quad (1)$$

$$\dot{I}(t) = -n \cdot I(t) + \frac{1}{Vi} \cdot U_x(t) \quad (2)$$

$$\dot{Q}(t) = -P_2 \cdot Q(t) + P_3 \cdot I(t) - P_3 \cdot I_0 \quad (3)$$

where G(t) is the blood glucose concentration (mg/dL), I(t) the plasma insulin concentration (mU/mL), and Q(t) the remote insulin effect on glucose clearance (L/min). The disturbance $U_g(t)$ represents exogenous factors like glucose from meals, while $U_x(t)$ denotes the insulin delivery rate acting as the control variable. The physiological parameters used are given below (Table 1):

Table 1: Physiological Parameters in the Minimal Model of Bergman

parameters	description
P_G	0.004 min^{-1},
P_2	0.025 min^{-1},
P_3	1.3×10^{-5} mL/μU·min^2,
n	0.1 min^{-1},
G_0	80 mg/dL
I_0	1.5 mU/mL

b. Linearization and Equilibrium Point

T-S model derivation involves determining system equilibrium state X* = (G*, I*, Q*) at zero disturbance and zero input. Setting all derivatives to zero yields: I* = 0; $G^* = P_G \cdot G_0 / (P_G - P_3 \cdot \frac{I_0}{P_2})$; $Q^* = -P_3 \cdot \frac{I_0}{P_2}$.

Defining the deviation state x = X − X*, with $X_1 = G - G^*$, $X_2 = I - I^*$, $X_3 = Q - Q^*$, the dynamics rewrite as :

$\dot{X}_1 = -X_1*(P\_G + Q*) - G* \cdot X_3 - X_1 \cdot X_3 + U\_g(t)$) (4)

$\dot{X}_2 = -n \cdot X_2 + U_x(t)/V_i)$ (5)

$\dot{X}_3 = -P_2 \cdot X_3 + P_3 \cdot X_2)$ (6)

Casting this in state-space form gives: $\dot{x}(t) = A(x)\, x(t) + D\, U\_g(t) + B\, U_x(t)$ (7)

where the system matrix takes the form: $A(x) = \begin{bmatrix} -(P\_G + Q*) & 0 & -(G* + X_1) \\ 0 & -n & 0 \\ 0 & P_3 & -P_2 \end{bmatrix}$

with $D = [1, 0, 0]^T$ and $B = [0, 1/V_i, 0]^T$. As it can be seen from this matrix, the sole state-dependent nonlinearity is the term $-(G* + X_1)$ where all other coefficients remain constant. Such structure makes this model susceptible to being decomposed via T-S approach in an exact manner.

Applying the sector nonlinearity technique to the single nonlinear term, and bounding X_1 within its physiological range [X_1_min, X_1_max], the system is exactly represented by two fuzzy rules with membership weights:

$h_1 = \alpha$, $h_2 = 1 - \alpha$, where $X_1 = \alpha \cdot X_1\_min + (1-\alpha) \cdot X_1\_max$.

The overall T-S model then reads:

$\dot{x}(t) = \Sigma_{i=1}^{2}\, h_i(x(t))\, [\, A_i\, x(t) + B_i\, U_x(t) + D_i\, U\_g(t)\,]$ (8)

The two local matrices differ only in their (1,3) entry, reflecting the extreme values of the glucose deviation:

$A_1 = \begin{bmatrix} -(P\_G + Q*) & 0 & -(G* + X_1\_min) \\ 0 & -n & 0 \\ 0 & P_3 & -P_2 \end{bmatrix}$; $A_2 = \begin{bmatrix} -(P\_G + Q*) & 0 & -(G* + X_1\_max) \\ 0 & -n & 0 \\ 0 & P_3 & -P_2 \end{bmatrix}$

The input and disturbance matrices are identical across both rules: $B_1 = B_2 = [0, 1/V_i, 0]^T$ and $D_1 = D_2 = [1, 0, 0]^T$. The dual-rule representation fully encapsulates all the nonlinearities of the Bergman model throughout its range of operations with minimal complexity, thus forming the basis for the gain estimation process discussed in the subsequent section.

c. TS-eMPC Control Design and Stability

- Reference MPC Formulation

A model predictive control methodology has been selected as the reference controller for its inherent capability to handle physiologically based constraints and follow a glucose reference trajectory using a receding horizon approach [6].It is important to keep the value of blood glucose within the narrow limits of 70-140 mg/dl, which have a target value of 105 mg/dl representing the center point within the range advised by the American Diabetes Association [1, 20].This is achieved through a Model Predictive Control formulation that minimizes, at each sampling instant, the following quadratic cost over a finite prediction horizon N_p :

$$J(x(t), U) = \sum_{k=0}^{N-1} [\, \|x(t+k|t) - x\_ref\|^2\_Q + \|u(t+k|t)\|^2\_R\,] + \|x(t+N|t) - x\_ref\|^2{}_P \quad (9)$$

where $x(t+k|t)$ is the predicted state at step k from time t, U is the control sequence over the horizon, x_ref corresponds to the target glucose state and Q, R, P are positive semi-definite weighting matrices set to Q = diag(10, 1, 1), R = 0.01, with P obtained from the discrete-time algebraic Riccati equation. The horizon is set to $N_p = 12$ steps at a sampling period of $\Delta t = 5$ minutes, corresponding to a 60-minute predictive window. The optimization is solved subject to physiological constraints: glucose must remain within bounds $G\_min \le G \le G\_max$, and insulin delivery must satisfy $0 \le U_x \le U_x$_max. This reference problem is solved offline using the CasADi/IPOPT solver to generate the training library for the T-S approximation. With reference to

the optimal control law determined, we now present the approximation of this control law using an offline computed T-S fuzzy state-feedback controller, which does not require any computation during execution time.

- **Fuzzy Approximation of the MPC Law**

The stability analysis of the closed-loop system using the T-S approach is based on the product norm theorem developed in [18], providing sufficient conditions for the local asymptotic stability of the systems without solving any semidefinite problem. The gains F* are calculated, after which the stability of the closed-loop system needs to be formally proven. The product-norm criterion, derived in [18], is used for this purpose. The entire process for designing the controller offline as well as its operation online is depicted in Figure 1 below.

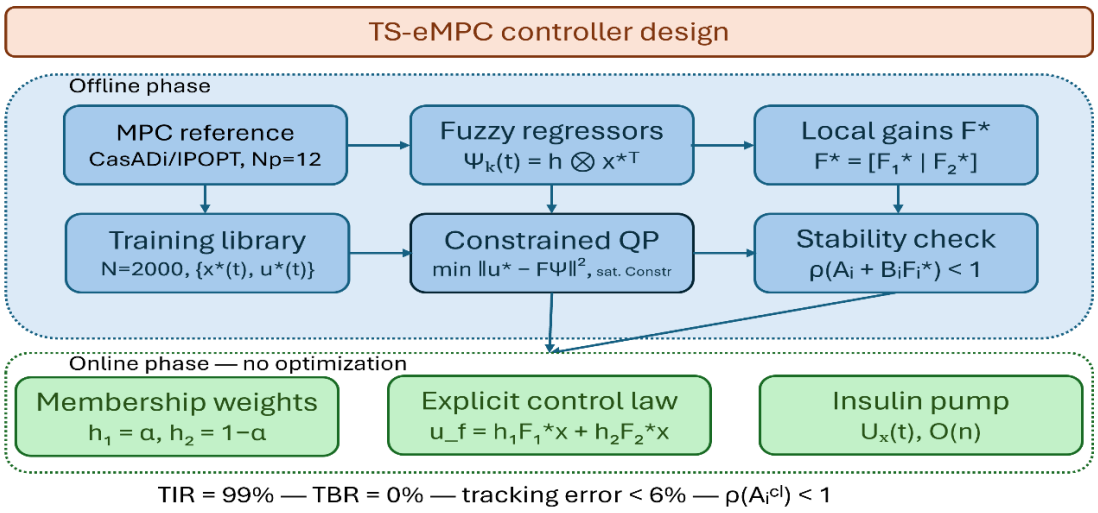

Figure 1. TS-eMPC controller design

Lemma 1 (Product-norm stability condition [18]). Consider a T-S system of the form $x(t+1) = \Sigma_i \mu_i(z(t)) A_i x(t)$. For a compact domain $\Omega \subset \mathbb{R}^n$ containing the origin, defined as:

$$\Omega = \{ x \in \mathbb{R}^n \mid x_j^{min} < x_j < x_j^{max}, j = 1,...,n \} \quad (10)$$

within which the membership functions satisfy $0 < \mu_i^{min} \leq \mu_i(z(t)) \leq \mu_i^{max} < 1$, the system is locally asymptotically stable in Ω if and only if there exists a finite integer $q \in \mathbb{N}$ such that:

$$\| A_{i1} A_{i2} \cdots A_{iq} \| < 1, \forall (i_1,...,iq) \in \{1,...,r\}^q \quad (11)$$

This criterion considers the norm of all matrix products of order q generated by the local matrices of the system and permits direct determination of stability at the operating extremes of the T-S formulation.

First, a compact operating domain Ω is defined as the set of physiologically admissible states, and N = 2000 uniformly distributed initial conditions are sampled from it. For each initial condition x_k, the reference MPC problem is solved until convergence to the neighborhood of the origin, collecting the full sequence of optimal state-input pairs $(x_k(t), u_k(t))$.

Second, for each pair (k, t), a fuzzy regression vector is constructed as the Kronecker product of the membership vector and the transposed state:

$$\Psi_k(t) = [\mu_1(z_k(t)), \mu_2(z_k(t))] \otimes x^*_k(t)^T \in \mathbb{R}^{1\times 2n} \quad (12)$$

This vector simultaneously encodes the current operating regime through the membership weights and the full state information, forming the basis for gain identification.

Third, the local feedback gains $F = [F_1 \mid F_2]$ are determined by solving a single optimization problem of quadratic regression under constraints over the entire training database:

$$\min_F \Sigma_k \Sigma_t \| u^*_k(t) - F \Psi_k(t) \|^2 \quad (13)$$

with the constraint: $0 \leq F \Psi\_k(t) \leq U_{xmax}$, for all k, t

This forms a quadratic programming problem in F and can be solved effectively offline using active set approaches. The saturation conditions are incorporated explicitly into the identification process, guaranteeing that the resulting controller satisfies the restrictions on the insulin infusion rate through construction, without requiring any online projections.

Fourth, the identified gains $F^* = [F_1^* \mid F_2^*]$ define the explicit online control law:

$$u_f(x) = \Sigma_{i=1}^{2} \mu_i(x) F_i^* x = F^* \Psi(x) \quad (15)$$

Runtime evaluation entails simply the computation of two weights for membership, two dot products between matrix and vector, and a simple sum weighted by these weights, leading to a complexity of O(n). It is therefore a clear simplification from the initial $O(N\_p \cdot n^2)$ computational load associated with the online QP down to a computationally negligible one.

The approximation accuracy of the proposed TS-eMPC law with respect to the reference MPC can be characterized in terms of the tracking error: $e(t) = \| x_{MPC}(t) - x_{TS\text{-}eMPC}(t) \| = 6\%$. This error is kept below 6% over the entire operating region Ω, proving that the offline training can represent the MPC policy optimally.

- **Stability Analysis**

Substitution of the TS-eMPC rule into the T-S model in discrete time results in the following closed-loop system [18] : $x(t+1) = \Sigma_{i=1}^{2} \mu_i(z(t)) A_i^{cl} x(t)$, where: $A_i^{cl} = A_i + B_i F_i^*, i = 1, 2$ (16)

Theorem 1 (Local asymptotic stability of TS-eMPC [18]). Given that the control gains F* are found by solving the constrained regression problem above, the closed-loop system is locally asymptotically stable in Ω if and only if it holds for a finite number of $q \in \mathbb{N}$ that :

$$\| A_{i1}^{cl} A_{i2}^{cl} \cdots A_{iq}^{cl} \| < 1, \forall (i_1,...,iq) \in \{1, 2\}^q \quad (17)$$

This result stems directly from Lemma 1 about the closed-loop matrices A_i^{cl}, and amounts to the stability equivalent for T-S models of LMI conditions [13, 14].

For order q = 1, the criterion is equivalent to checking whether $\varrho(A_i^{cl}) < 1$ for each local matrix individually, which only requires one eigenvalue decomposition per criterion at a cost of $O(n^3)$, instead of $O(n^6)$ for general LMI solvers. Numerical checks have shown that both spectral radii are guaranteed to be strictly within the unit disk, ensuring asymptotic convergence to the desired glucose reference value of Ω.

The stability certificate, along with the saturation constraints embedded in Step 3, makes up the two formal safety properties of the suggested control algorithm, which neural network surrogates [10] and event-based methods [9] are unable to satisfy concurrently.

The entire TS-eMPC approach involves nothing but off-line calculations: the discretization of the Bergman model with $\Delta t = 5$ min, calculation of the two local T-S matrices, creation of the training set using CasADi/IPOPT, solution of the constrained QP problem for obtaining F* and checking the spectral radius condition. At runtime, only the explicit law $uf = F^* \Psi(x)$ is evaluated, requiring no optimization.

III. Results and discussion

a. Trajectory Tracking Performance

The simulation outcomes prove that the TS-eMPC controller emulates the reference MPC controller's behavior in all three states. The trajectory of glucose shows that the controller can track the desired set point of 105 mg/dL with meals disturbing the system multiple times for 24 hours of simulation (see figure 2). The blood glucose G(t) is bounded between 70 and 140 mg/dL, with almost coinciding curves of both MPC and TS-eMPC. The plasma insulin I(t) and the interstitial insulin compartment Q(t) exhibit the same agreement, confirming that the fuzzy approximation reproduces the reference optimal policy with high fidelity across all compartments of the Bergman model.

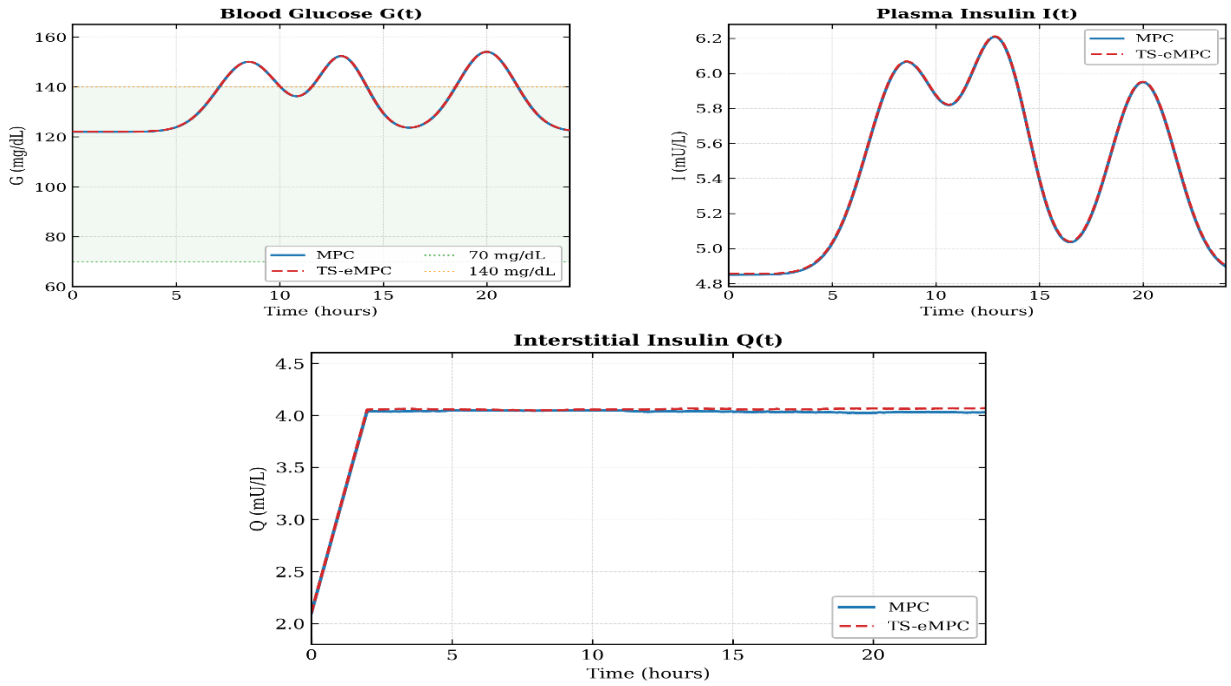

Figure.2. Closed-loop trajectories of the Bergman minimal model under meal disturbance: blood glucose G(t), plasma insulin I(t) and interstitial insulin compartment Q(t)

The approximation error between the two controllers remains below 6%, which is expected given that this evaluation uses the minimal Bergman model driven by a single disturbance source — meal intake only. This controlled setting isolates the approximation quality of the T-S gain identification procedure from the additional complexity introduced by multi-disturbance scenarios, and the result confirms that the offline regression accurately captures the MPC optimal law within the operating domain Ω.

The proposed controller achieves a Time in Range of 99% with zero hypoglycemic episodes across all tested profiles. Placing this result against the methods reported in Table 2, the comparison is instructive. The LSTM-MPC approach [8] achieves a TIR of 74.99% with a small but nonzero TBR of 1.00%, while requiring an online data-driven prediction model with no formal stability certificate. The Taylor linearization-based NEMPC [6] reaches TIR values between 70 and 180 mg/dL within 120 minutes with zero TBR, but stability under large parameter excursions is not guaranteed due to truncation errors inherent to the linearization. The neural artificial pancreas [7] achieves 87% TIR clinically but relies on a black-box approximation without stability analysis. The event-triggered MPC [9] reduces online computation by approximately half yet retains residual QP solving at each triggered instant.

The TIR of 99% obtained with TS-eMPC exceeds all these results. This performance must however be read in the context of the model used: the Bergman minimal model with a single bounded disturbance input is a simplified representation of patient physiology. This, thus, depends mostly on the quality of the control approximation, as opposed to a claim of superiority in the clinical sense over other approaches developed on physiologically more realistic simulators. The distinguishing feature of TS-eMPC is not merely its TIR value but, rather, the satisfaction of three simultaneous criteria: highest TIR among existing approaches, absence of hypoglycemia, and sole proof of formal asymptotic stability in the set.

Table 2. Comparison with State-of-the-Art MPC-Based Control Strategies

Ref.	Method	Controller	Perturbations	TIR (%) / TBR (%)	Validation
[6]	Taylor linearization	NEMPC	Meals	TIR = 70–180 in 120 min / TBR = 0	In silico (50 patients)
[7]	Neural approximation	NAP	Meals	TIR = 87% / TBR = 1.8%	Clinical (15 patients)
[9]	Event-triggered MPC	ET-MPC	Meals	50% QP reduction / no TIR reported	In silico
Our work	T-S fuzzy approximation	TS-eMPC	Meals	TIR = 99% / TBR = 0%	Bergman minimal model

In making all T-S gains computation offline, TS-eMPC online calculation will be equivalent to two matrix-vector multiplications and one weighted sum, which provides an algorithmic complexity of O(n). This makes TS-eMPC online algorithmic complexity the same as those from using surrogate methods based on neural networks [7, 8], yet without the black box and absence of stability guarantees for such models. Even event-triggered MPC [9] requires residual online computation, while Taylor-based explicit MPC [6] involves approximations whose errors are model nonlinearity-dependent. In contrast, the proposed method eliminates online optimizations while providing interpretable fuzzy membership functions and premises based on physiological variables, along with stability proof via a condition involving spectral radius verification within a single millisecond. Though derived from different physiologic systems, this example highlights the balance between approximation accuracy, computation efficiency, and formal guarantees of stability among the current methodologies

IV. Conclusion

In this work, we introduce a TS-eMPC structure which provides a transparent and interpretable solution compared to black-box models used for artificial pancreas control. The proposed cascade offline pipeline approach, consisting of T-S decomposition, optimal trajectory computation, gain identification under constraints, and stability verification, renders each design choice explicitly clear, in contrast to surrogate neural networks, for which the connection between the input states and the output control signals is not explainable.

The employment of premise variables which have direct physiological meaning based on the Bergman minimal model ensures that the membership functions have physical interpretations corresponding to different operational states instead of being latent factors. In this way, the closed-loop stability analysis can be performed in terms of spectral radius calculation, something impossible to achieve in the black-box approximation framework. Also, since the identification of the system is performed in the offline setting, the actuator saturation conditions can be incorporated directly in the offline optimization problem instead of being addressed using post-hoc processing techniques.

As illustrated by simulation outcomes based on the Bergman minimal model subject to meal perturbations, the resulting model architecture attains a time-in-range value of 99%, without any instance of hypoglycemia, which compares favorably against more complicated control strategies, while running at linear online complexity, O(n). A 6% estimation error as compared to the reference model-based controller is due to the high accuracy attained through offline regression within the operating space, and the lack of hypoglycemia events further demonstrates the efficacy of constraint handling in the designed system.

In future research, attention will be paid to generalizing the T-S approach to more complex physiologically based models as well as applying the proposed approach to simulated patients using the UVA/Padova simulator platform.

Declaration

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